H. Algebra 2

Unit 3 Review

Name: _____ **Date:** _____ **Hour:** ___

Module 5 – Polynomials Functions

5.1 Graphing Cubic Functions

Describe, in words, the transformations applied to the graph of $f(x) = x^3$ to produce the graph of $g(x)$.

1. $g(x) = -3(x+1)^3 - 9$

Left 1, down 9, reflect over x-axis, ver str of 3

2. $g(x) = \frac{1}{4}(-x)^3 + 5$ Up 5, reflect over y

Vertical shrink of 1/4.

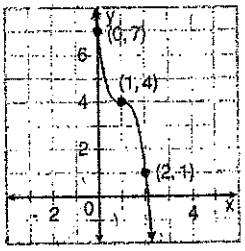
3. $g(x) = (2(x-1))^3$

Right 1, Horizontal shrink of 1/2

4. $g(x) = \left(\frac{1}{3}(x+4)\right)^3 - 7$ Left 4, down 7, horizontal stretch of 3.

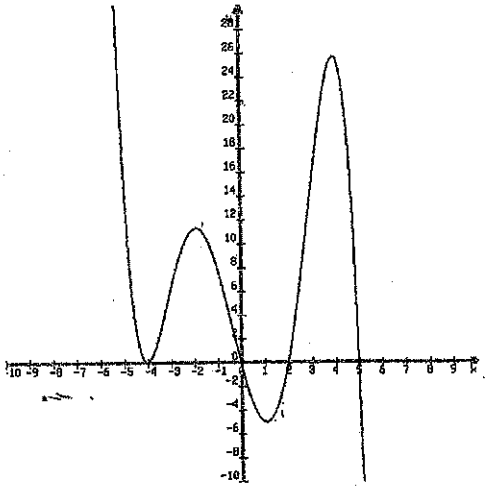
5. Write the equation of the form $(x) = a\left(\frac{1}{b}(x-h)\right)^3 + k$ for the graph.

$y = -3(x-1)^3 + 4$



5.2 Graphing Polynomial Functions

Given the graph, identify the following:



6. End Behavior. as $x \to -\infty$, $y \to \infty$

as $x \to \infty$, $y \to -\infty$

7. Even or Odd degree? odd **8.** Positive or negative leading coeff? —

9. Zeros and multiplicities. **10.** Number of turning Points 4

−4, 0, 2, 5

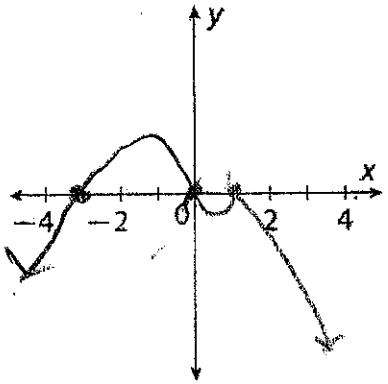
all multiplicity of 1

11. # of Global Max 0 **12.** # of Local Max 2

13. # of Global Min 0 **14.** # of Local Min 2

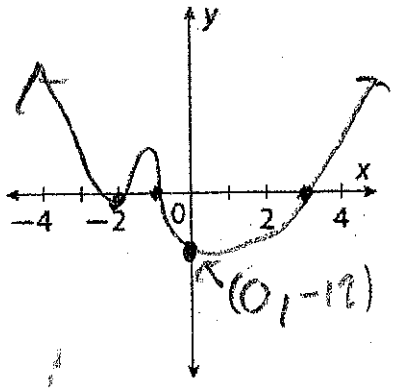
Use the end behavior, x-intercepts, and y-intercept to sketch the graph of the function.

15. $g(x) = -x(x-1)^2(x+3)$



16. $g(x) = (x-3)(x+1)(x+2)^2$

$-3(1)(4)$



5 14 23

Module 6 – Polynomials

6.1 Adding and Subtracting Polynomials

Add or Subtract. Write your answers in standard form.

1. $(x - 4 + 6x^2 + 8x^3) + (17x - 9x^2 + 6)$

$8x^3 - 3x^2 + 18x + 2$

2. $(5x^3 + 7x^2 - 10x^4 + x^{12}) + (3x^{12} + 4x^2 + 1)$

$-2x^{12} - 10x^4 + 5x^3 + 3x^2 + 1$

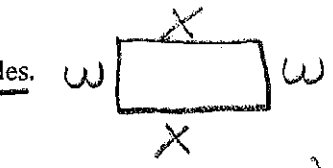
3. $(8x^5 + x^3 - x) + (x + 7x^5)$

$x^5 + x^3 - 2x$

4. $(12x + 11x^2 - 10x^3 + 4) + (-9x^3 - 14x + 5)$

$-19x^3 + 11x^2 - 2x + 9$

5. A rectangular field has a perimeter of $(3x^3 - 12x^2 + 10x - 75)$ miles and a length of x miles.



a) Write an equation for the width of the field in terms of the length.

$w = 1.5x^3 - 6x^2 + 15x - 37.5$

$P = 2(x + w)$

$P = 3x^3 - 12x^2 + 10x - 75$

$2(x + w) = 3x^3 - 12x^2 + 10x - 75$

$x + w = 1.5x^3 - 6x^2 + 5x - 37.5$

$-x \qquad -x$

b) Find the width of the field when the length is 5 miles.

$w = 20$ miles

6.2 Multiplying Polynomials

Multiply. Write your answers in standard form.

1. $(x^2 - 2x + 6)(x^3 - 5x)$

$x^5 - 5x^3 - 2x^4 + 10x^2 + 6x^3 - 30x$

$x^5 - 2x^4 + x^3 + 10x^2 - 30x$

2. $(x^4 + 2x^2)(4x - 3)$

$4x^5 - 3x^4 + 8x^3 - 6x^2$

3. $(2xy - 4x)(y^2 - 3xy + y)$

$2xy^3 - 6x^2y^2 + 2xy^2 - 4xy^2 + 12x^2y - 4xy$

$-6x^2y^2 - 2xy^3 - 2xy^2 + 12x^2y - 4xy$

4. $(3p^2 + 4p + 5)(2 - p)$ $6p^2 - 3p^3 + 8p - 4p^2 + 10 - 5p$

$-3p^3 + 2p^2 + 3p + 10$

6.3 The Binomial Theorem

Expand using the Binomial Theorem

1. $(x - 2)^5$ $1(x)^5 - 5(x)^4(-2)^1 + 10x^3(-2)^2 + 10x^2(-2)^3 + 5x(-2)^4 - 2^5$

$x^5 + 10x^4 + 40x^3 - 80x^2 + 80x - 32$

2. $(3m - n)^4$ $1(3m)^4 + 4(3m)^3(-n) + 6(3m)^2(-n)^2 + 4(3m)(-n)^3 + 1(-n)^4$

$81m^4 - 108m^3n + 54m^2n^2 - 12mn^3 + n^4$

3. $(x + 2y)^6$ $1(x)^6 + 6(x)^5(2y)^1 + 15(x)^4(2y)^2 + 20(x)^3(2y)^3 + 15x^2(2y)^4 + 6x(2y)^5 + (2y)^6$

$x^6 + 12x^5y + 60x^4y^2 + 160x^3y^3 + 240x^2y^4 + 192xy^5 + 64y^6$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

Find the specific term in each binomial expansion.

4. $(x - 2)^5$; 4th term $_5C_3$

$10(x^3)(-2)^2$

$-80x^2$

5. $(3x - 1)^4$; 3rd term $_4C_2$

$6(3x)^2(-1)^2$

$54x^2$

6. $(3 + x)^7$; 5th term $_7C_4$

$35(3)^3(x)^4$

$945x^4$

6.4 Factoring Polynomials

Factor completely.

1. $10x^3 - 4x^2 - 15x + 6$
$2x^2(5x-2) - 3(5x-2)$
$(2x^2-3)(5x-2)$

2. $6x^3 + 3x^2 - 4x - 2$
$3x^2(2x+1) - 2(2x+1)$
$(3x^2-2)(2x+1)$

3. $125x^3 - 64$
$(5x-4)(25x^2+20x+16)$

4. $10x^4 + 9x^2 - 40$
$(10x^4 - 16x^2)(25x^2 - 40)$
$2x^2(5x^2-8) + 5(5x^2-8)$
$(2x^2+5)(5x^2-8)$

5. $15x^3 + 20x^2 - 3x - 4$
$5x^2(3x+4) - 1(3x+4)$
$(5x^2-1)(3x+4)$

6. $100x^4 - 25$
$(10x^2-5)(10x+5)$

7. $2x^4 + 7x^3 + 5x^2$
$x^2(2x^2+7+5)$
$x^2(2x^2+5)(2x+5)$
$x^2(2x+5)(x+1)$

8. $x^3 + x^2 - 12x$
$x(x^2+x-12)$
$x(x+4)(x-3)$

9. $128x^3 + 54$ $64=4^3$ $27=3^3$
$2(64x^3+27)$
$2(4x+3)(16x^2-12x+9)$

6.5 Dividing Polynomials

Use long division to divide the polynomials. Write your answers as *dividend* = (*divisor*)(*quotient*) + *remainder*.

1. $(5x^4 - 42x^3 - 77x^2 - 5x + 10) \div (5x + 3)$
$x^3 - 9x^2 + 10x - 5 - \frac{5}{5x+3}$
$-(15x^4 + 9x^3)$
$-45x^3 - 77x^2$
$-(-45x^3 - 27x^2)$
$-50x^2 - 5x$
$-(-50x^2 - 30x)$
$25x + 10$
$-(-25x - 15)$
$+5$

2. $(4x^3 + 2x^2 + 3x + 5) \div (x^2 + 3x + 1)$
$4x - 10 + \frac{29x+15}{x^2+3x+1}$
$-(4x^3 + 12x^2 + 4x)$
$-10x^2 - x + 5$
$-(-10x^2 - 30x - 10)$
$+29x + 15$
$4x^3+2x^2+3x+5 = x^2+3x+1\,(x^2+3x+1) + 29x + 15 - 10x^2 - x + 5 + (-10x^2 - 30x - 10) + 29x + 15$

3. $(9x^4 - 9x^3 + 3) \div (9x - 9)$

4. $(2x^4 + x^3 - 31x^2 - 27x + 25) \div (2x + 7)$

Use synthetic division to divide the polynomials. Write your answers as *dividend* = (*divisor*)(*quotient*) + *remainder*.

5. $(x^4 - 2x^3 + 5x - 15) \div (x - 2)$
2 | 1 −2 0 5 −15
| 1 2 0 0 10
1 0 0 5 | −5
$(x^4-2x^3+5x-15) = (x-2)(x^3+5) + -5$

6. $(x^4 + 15x^3 + 61x^2 + 43x + 25) \div (x + 8)$
−8 | 1 15 61 43 25
| 1 −8 −56 −40 −24
1 7 5 3 | 1
$x^4+15x^3+61x^2+43x+25 = (x+8)(x^3+7x^2+5x+3) + 1$

Use synthetic substitution to evaluate $p(x)$ for the given value.

7. $p(x) = -2x^4 + x^3 - 15x - 10; x = 2$
2 | −2 1 0 −15 −10
↓ −4 −6 −12 −54
−2 −3 −6 −27 | −64

8. $p(x) = -x^5 - 4x^2 + 1; x = -4$
−4 | −1 0 0 −4 0 1
↓ 4 −16 64 −240
−1 4 −16 60 | −239

Determine whether the given binomial is a factor of $p(x)$. If it is, completely factor $p(x)$.

9. $(x - 3); p(x) = x^3 + 3x^2 - 34x + 48$
3 | 1 3 −34 48
| 1 3 −18 −48
1 6 −16 | 0
$(x-3)(x^2+6x-16)$
$(x-3)(x+8)(x-2)$

10. $(x + 5); p(x) = -2x^3 + 6x^2 - x$
−5 | −2 6 −1 0
↓ 10 −80 405
−2 16 −81 | 405
not a factor

Module 7 – Polynomial Equations

7.1 Finding Rational Solutions of Polynomial Equations

Solve each polynomial equation by factoring.

1. $4x^3 + x^2 - 4x - 1 = 0$

$x^2(4x+1) - 1(4x+1)$

$(x^2-1)(4x+1)$

$(x-1)(x+1)(4x+1)$

$x-1=0$ $x+1=0$ $4x+1=0$

$x=1$ $x=-1$ $x=-1/4$

2. $x^5 - 2x^4 - 24x^3 = 0$

$x^3(x^2-2x-24)$

$x^3(x-6)(x+4)=0$

$x^3=0$ $x-6=0$ $x+4=0$

$x=0$ $x=6$ $x=-4$

3. $3x^5 + 18x^4 - 21x^3 = 0$

$3x^3(x^2+6x-7)=0$

$3x^3(x+7)(x-1)=0$

$3x^3=0$ $x+7=0$ $x-1=0$

$x=0$ $x=-7$ $x=1$

4. $-x^4 + 2x^3 + 8x^2 = 0$

$-x^2(x^2-2x-8)=0$

$-x^2(x+2)(x-4)$

$-x^2=0$ $x+2=0$ $x-4=0$

$x=0$ $x=-2$ $x=4$

List all possible rational zeros of the function, then write the function in factored form.

5. $f(x) = x^3 + 3x^2 + 3x + 1$ Possibles ± 1

-1 | 1 3 3 1

	1	-1	-2	-1
	1	2	1	0

$(x+1)(x^2+2x+1)$

$f(x) = (x+1)(x+1)(x+1)$

$f(x) = (x+1)^3$

6. $f(x) = x^3 + 5x^2 - 8x - 48$ Possibles $\pm 1, 2, 3, 4, 6, 8, 12, 16, 24, 48$

m / n	1	5	-8	-48
3	1	8	16	0

$(x-3)(x^2+8x+16)$

$f(x) = (x-3)(x+4)^2$

List all possible rational roots of each equation, then find the actual roots.

7. $x^3 + 10x^2 + 17x = 28$ Roots $\pm 1, 2, 4, 7, 14, 28$

$x^3 + 10x^2 + 17x - 28$

1 | 1 10 17 -28

	1	1	11	28
	1	11	28	0

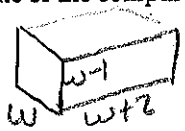
$(x-1)(x^2+11x+28)$

$f(x) = (x-1)(x+7)(x+4)$

8. $3x^3 + 10x^2 - 27x = 10$

9. An engineer is designing a storage compartment in a spacecraft. The compartment must be 2 meters longer than it is wide, and its depth must be 1 meter less than its width. The volume of the compartment must be 8 cubic meters.

a) Write an equation to model the volume of the compartment.



$V = w(w-1)(w+2)$

$V = w^3 + w^2 + 2w$

$8 = w^3 + w^2 + 2w$

$0 = w^3 + w^2 + 2w - 8$

b) List all possible rational roots.

$\pm 1, 2, 4, 8$

c) Use synthetic division to find the roots of the polynomial equation. Are the roots all rational numbers? no

1 | 1 1 -2 -8

	1	1	2	-4
	1	2	-4	-12

± 2 | 1 1 -2 -8

	1	2	6	8
	1	3	4	0

$(x-2)(x^2+3x+4)$

$x=2$

$x = \frac{-3 \pm \sqrt{9-4(1)(4)}}{2}$

$x = \frac{-3 \pm \sqrt{-7}}{2}$

$x = \frac{-3 \pm i\sqrt{2}}{2}$

$(2-3i)(2+3i)$

7.2 Finding Complex Solutions of Polynomial Equations

Write the simplest polynomial function with the given roots.

1. 1, 4, and -3 $(x-1)$

$(x-1)(x-4)(x+3)$

$(x^2-5x+4)(x+3)$

$x^3+3x^2-5x^2-15x+4x+12$

$x^3-2x^2-11x+12$

2. $\frac{1}{2}$, 5, and -2 $(x-\frac{1}{2})(x-5)(x+2)$

$(2x-1)(x^2-3x-10)$

$2x^3-6x^2-20x-x^2+3x+10$

$2x^3-7x^2-17x+10$

3. $2i$, $\sqrt{3}$, and 4, $-\sqrt{3}, -2i$

$(x+2i)(x+2i)(x-\sqrt{3})(x+\sqrt{3})(x-4)$

$(x^2+2ix-2ix-4i^2)(x^2+\sqrt{3}x-\sqrt{3}x-\sqrt{9})(x-4)$

$(x^2+4)(x^2-3)(x-4)$

$(x^4-3x^2+4x^2-12)(x-4)$

$(x^4+x^2-12)(x-4)$

$x^5-4x^4+x^3-4x^2-12x+48$

4. $\sqrt{2}$, -5, and $-3i$, $3i$, $-\sqrt{2}$

$(x+5)(x-\sqrt{2})(x+\sqrt{2})(x+3i)(x-3i)$

$(x+5)(x^2+\sqrt{2}x-\sqrt{2}x-\sqrt{4})(x^2-3ix+3ix-9i^2)$

$(x+5)(x^2-2)(x^2+9)$

$(x^3-2x+5x^2-10)(x^2+9)$

$x^5+9x^3-2x^3-18x+5x^4+45x^2-10x^2-90$

$P(x)=x^5+5x^4+7x^3+35x^2-18x-90$

Solve each equation by finding all roots.

5. $x^4-2x^3-14x^2-2x-15=0$ RR

6. $x^4-16=0$

7. $x^4+4x^3+4x^2+64x-192=0$

	1	4	4	64	−192
1	1	5	9	73	no
2	1	6	16	96	0

$(x-2)(x^3+6x^2+16x+96)$

	1	6	16	96
−2	1	4	8	80
−3	1	3	7	no
−4	1	2	8	96
−6	1	0	16	0

$\pm1, \pm2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 64, 96, 192$

$(x-2)(x+6)(x^2+16)=0$

$x-2=0$ $x+6=0$ $x^2+16=0$

$x=2$ $x=-6$ $\sqrt{x^2}=\sqrt{-16}$

$x=\pm4i$

8. $x^3-64=0$ x^3-4^3

$(x-4)(x^2+4x+16)=0$

$x-4=0$

$x=4$

$x^2+4x+16=0$

$x=\frac{-4\pm\sqrt{4^2-4(1)(16)}}{2(1)}$

$x=\frac{-4\pm\sqrt{16-64}}{2}$

$x=\frac{-4\pm\sqrt{-48}}{2}$

$x=\frac{-4\pm4i\sqrt{3}}{2}$

$x=-2\pm2i\sqrt{3}$

9. An electrical circuit is designed such that its output voltage, V, measured in volts, can be either positive or negative. The voltage of the circuit passes through zero at $t = 1, 2$, and 7 seconds. Write the simplest polynomial describing the voltage $V(t)$. $= (t-1)(t-2)(t-7)$

$$(a^3-b^3)=(a-b)(a^2+ab+b^2)$$